Solid Rocket Motor Preliminary Design Sizing and Performance Analysis

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November 30, 2025

Problem Statement

Design a solid rocket motor using 18% Al, 71% AP, 11% HTPB. Perform preliminary sizing of the BATES grain and nozzle to estimate performance at the stated initial conditions.

Table 1: Design Inputs (project brief)

Parameter	**Symbol**	**Value**
Mission ΔV	ΔV	550 m/s
Initial mass limit	m_0	45.36 kg
Initial chamber pressure	P_c	12.410 MPa
Design thrust	F	2224.1 N
Ambient pressure	P_a	0.075 84 MPa
Grain geometry	–	BATES (outer inhibited)
Length-to-diameter	L/D	8.5
Expansion ratio	ε	21.297
Combustion temperature	T_c	3400 K
Molecular weight	MW	31 g/mol
Propellant density	ρ_p	1800 kg/m^3
Specific heat ratio	γ	1.18
Burning law exponent	n	0.30
Burning law coefficient	a	0.399 (cm/s)/(MPa)$^{0.3}$

Key Relations Used

$$\dot{r} = a\,P_c^n$$

$$R = \frac{R_u}{MW}, \quad R_u = 8314.462 \text{ J/(kmol} \cdot \text{K)}$$

$$c^* = \sqrt{\frac{RT_c}{\gamma}}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A}{A^*} = \frac{1}{M}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{P_e}{P_c} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{-\frac{\gamma}{\gamma-1}}$$

$$C_F = \sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} + \frac{P_e - P_a}{P_c}\frac{A_e}{A_t}$$

$$A_t = \frac{F}{P_c\,C_F}, \qquad d = \sqrt{\frac{4A}{\pi}}$$

Table 2: Section A1: Solid Propellant Grain Sizing and Geometry

Quantity	**Value**
Grain length L	$0.777\,\mathrm{m}$
Grain volume V_p	$4.9075 \times 10^{-3}\,\mathrm{m}^3$
Grain mass m_p	$8.8335\,\mathrm{kg}$
Inner port diameter d_i	$18.2835\,\mathrm{mm}$
Outer diameter d_o	$91.48\,\mathrm{mm}$
Web thickness ω_o	$36.604\,\mathrm{mm}$
Volumetric loading fraction (VLF)	96%
Port-to-throat area ratio	2.511

All Work / Step-by-Step

Unit Conversions and Constants

$$a = 0.399\ \frac{\text{cm}}{\text{s}\cdot\text{MPa}^{0.3}} = 0.00399\ \frac{\text{m}}{\text{s}\cdot\text{MPa}^{0.3}}, \qquad P_c = 12.410\ \text{MPa} = 12.410\times 10^6\ \text{Pa}$$

$$MW = 31\ \text{kg/kmol}, \quad R = \frac{8314.462}{31} = 268.20845\ \text{J/(kg K)}$$

Burn Rate

$$(12.410)^{0.3} = 2.126237, \quad \dot{r} = 0.00399\times 2.126237 = 8.4938\times 10^{-3}\ \text{m/s}$$

Characteristic Velocity

$$\sqrt{\frac{RT_c}{\gamma}} = \sqrt{\frac{268.20845\times 3400}{1.18}} = 879.4276,$$

$$\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = 1.68378,$$

$$c^* = 879.4276\times 1.68378 = 1481.402\ \text{m/s}$$

Propellant Mass and Volume

$$\Delta V = 550\ \text{m/s}, \quad m_p = m_0\left(e^{\Delta V/c}-1\right)/e^{\Delta V/c}$$

$$m_p = 45.36\cdot\left(e^{550/2539.336}-1\right)/e^{550/2539.336} = \boxed{8.8335\ \text{kg}}$$

$$V_p = \frac{m_p}{\rho_p} = \frac{8.8335}{1800} = \boxed{4.907488\times 10^{-3}\ \text{m}^3}$$

Delivered Exhaust Velocity and Specific Impulse

$$c = C_F \cdot c^* = 1.71419 \cdot 1481.36 = \boxed{2539.336 \text{ m/s}}$$

$$I_{sp} = \frac{c}{g_0} = \frac{2539.336}{9.80665} = \boxed{258.9 \text{ s}}$$

Thrust Coefficient

Note: Exit pressure P_e and associated flow properties were obtained using the Virginia Tech Rocket Propulsion Calculator under initial operating conditions.

$$C_{F,\text{iso}} = 1.7058, \quad \frac{P_e - P_a}{P_c}\frac{A_e}{A_t} = -0.00934, \quad C_F = 1.71419$$

Throat/Exit Areas and Diameters

$$A_t = \frac{F}{P_c C_F} = \frac{2224.1}{(12.410 \times 10^6) \times 1.71419} = 1.04550 \times 10^{-4} \text{ m}^2,$$

$$d_t = \sqrt{\frac{4A_t}{\pi}} = 11.538 \text{ mm},$$

$$A_e = \varepsilon A_t = 2.2266 \times 10^{-3} \text{ m}^2, \quad d_e = 53.245 \text{ mm}$$

Mass Flow

$$\dot{m} = \frac{P_c A_t}{c^*} = \frac{12.410 \times 10^6 \cdot 1.04550 \times 10^{-4}}{1481.402} = 0.87583 \text{ kg/s}$$

Table 3: Section B1: Nozzle Sizing and Exit Flow Properties

Quantity	**Value**
Throat diameter d_t	11.538 mm
Exit diameter d_e	53.245 mm
Initial Kn Kn_0	547.9
Final Kn Kn_{final}	1936.47
Maximum Kn Kn_{max}	1936.47
Optimum nozzle length (conical) L_{conical}	77.826 mm
Optimum nozzle length (bell, 80%) L_{bell}	62.261 mm

Initial values ($t = 0$)

Initial burning surface area (BATES geometry, adjusted):

$$r_i = \frac{d_{i,0}}{2} = \frac{0.0182835}{2} = 0.00914175 \text{ m},$$
$$r_o = \frac{d_o}{2} = \frac{0.09148}{2} = 0.04574 \text{ m}$$

$$S_b(0) = 2\pi r_i(17r_o) + 2\pi(r_o^2 - r_i^2)$$

$$\begin{aligned} S_b(0) &= 2\pi(0.00914175)(17 \cdot 0.04574) + 2\pi\left((0.04574)^2 - (0.00914175)^2\right) \\ &= 2\pi(0.007105) + 2\pi(0.002009) \\ &= 0.044643 + 0.012644 \\ S_b(0) &= \boxed{0.0572872 \text{ m}^2} \end{aligned}$$

$$A_{\text{port},0} = \frac{\pi d_{i,0}^2}{4} = \frac{\pi(0.0182835)^2}{4} = 2.6255 \times 10^{-4} \text{ m}^2$$
$$\frac{A_{\text{port},0}}{A_t} = \frac{2.6255 \times 10^{-4}}{1.04550 \times 10^{-4}} = 2.511, \qquad \text{Kn}_S(0) = \frac{0.0572872}{1.04550 \times 10^{-4}} = \boxed{547.9}$$
$$\dot{m}_{\text{gen}}(0) = \rho_p \, S_b(0) \, \dot{r} = 1800 \cdot 0.0572872 \cdot 8.4938 \times 10^{-3} = \boxed{0.8752 \text{ kg/s}}$$

At analytic burnout ($d_i \to d_o$)

$$S_{b,\text{final}} = \pi D_o(L - D_o + D_i) = \pi(0.09148)(0.776 - 0.09148 + 0.0182835) = \boxed{0.20198 \text{ m}^2}$$

Note: Since the grain exhibits progressive burning, the final Kn value corresponds to the maximum Kn. Therefore, $\text{Kn}_{\text{max}} = \text{Kn}_{\text{final}}$.

$$\text{Kn}_{\text{max}} = \frac{S_{b,\text{final}}}{A_t} = \boxed{1936.47}$$

$$\dot{m}_{\text{gen}}(t_b) = \rho_p \, S_b(t_b) \, \dot{r} = 1800 \cdot 0.20198 \cdot 8.4938 \times 10^{-3} = \boxed{3.0835 \text{ kg/s}}$$

Theoretical Burn Time Calculation

The pressure during burn is given by:

$$P_c(r) = \left(\frac{c^* \rho_p a S_b}{A_t}\right)^{\frac{1}{1-n}}$$

$$P_c(r) = \left(\frac{1481.36 \cdot 1800 \cdot 6.323 \times 10^{-5} \cdot 0.143}{1.0455 \times 10^{-4}}\right)^{\frac{1}{1-0.3}} = \boxed{45{,}664{,}435 \text{ Pa}}$$

Now integrate to find total burn time:

$$T_b = \int_{r_0}^{r_o} \frac{dr}{a \cdot P_c(r)^n} = \int_{0.00914175}^{0.04574} \frac{dr}{6.323 \times 10^{-5} \cdot (45{,}664{,}435)^{0.3}} = \boxed{2.93 \text{ s}}$$

Reference steady throat mass flow (for comparison)

$$\dot{m}_{\text{nozzle,steady}} = \frac{P_c A_t}{c^*} = 0.87583 \text{ kg/s}.$$

Optimal web thickness for maximum S_b

$$\frac{dS_b}{dx} = 0 \Rightarrow x = \frac{1}{6}(L - 2D_i) = \boxed{\text{x} = 370.586 \text{ mm}}$$

Optimum nozzle lengths

Assume $\alpha = 15°$, $D_e = 0.053245$ m, $D_t = 0.011538$ m.

$$L_{\text{conical}} = \frac{D_e - D_t}{2\tan(\alpha)} = \frac{0.053245 - 0.011538}{2\tan(15°)} = \boxed{77.826 \text{ mm}}$$

$$L_{\text{bell}} = 0.8 \cdot L_{\text{conical}} = \boxed{62.261 \text{ mm}}$$

Initial and Nozzle Properties (Virginia Tech Calculator)

Table 4: Initial Nozzle Properties (Virginia Tech Calculator)

Quantity	**Symbol**	**Value**
Exit pressure	P_e	0.062 104 3 MPa
Exit Mach number	M_e	3.71724
Exit temperature	T_e	1515.42 K
Speed of sound at exit	a_e	692.518 m/s
Exit velocity	V_e	2574.25 m/s
Exit density	ρ_e	0.152 806 kg/m^3

Table 5: Final/Maximum Kn Nozzle Properties (Virginia Tech Calculator)

Quantity	**Symbol**	**Value**
Chamber pressure	P_c	75.339 72 MPa
Exit pressure	P_e	0.377 027 8 MPa
Exit Mach number	M_e	3.71724
Exit temperature	T_e	1515.42 K
Speed of sound at exit	a_e	692.518 m/s
Exit velocity	V_e	2574.25 m/s
Exit density	ρ_e	0.927 67 kg/m^3

B2: Ansys Simulation and Nozzle Visualizations

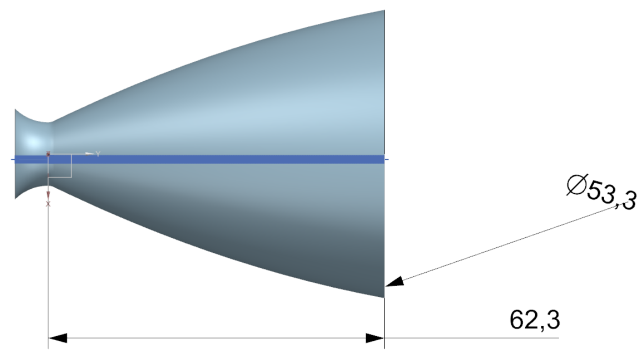

Figure 1: Bell nozzle geometry with dimensions.

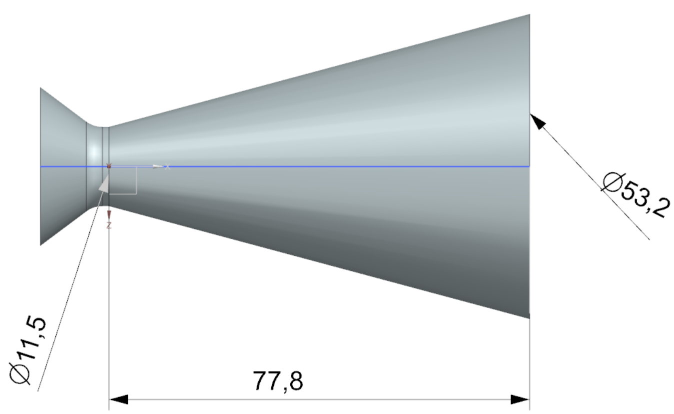

Figure 2: Conical nozzle geometry with dimensions.

Conical Nozzle – Line Plots

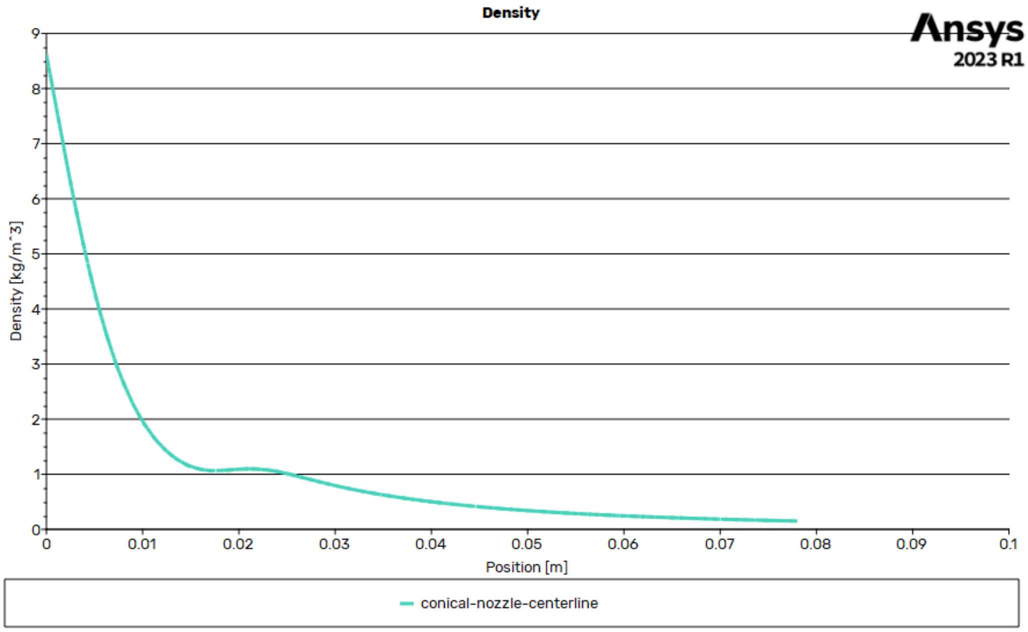

Figure 3: Density distribution along the centerline of the conical nozzle.

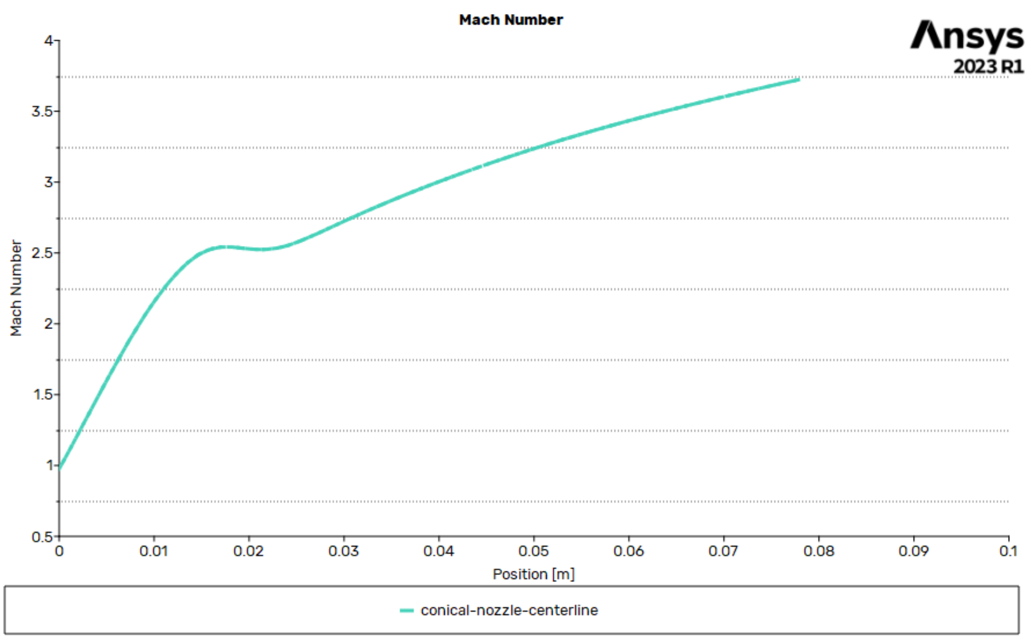

Figure 4: Mach number distribution along the centerline of the conical nozzle.

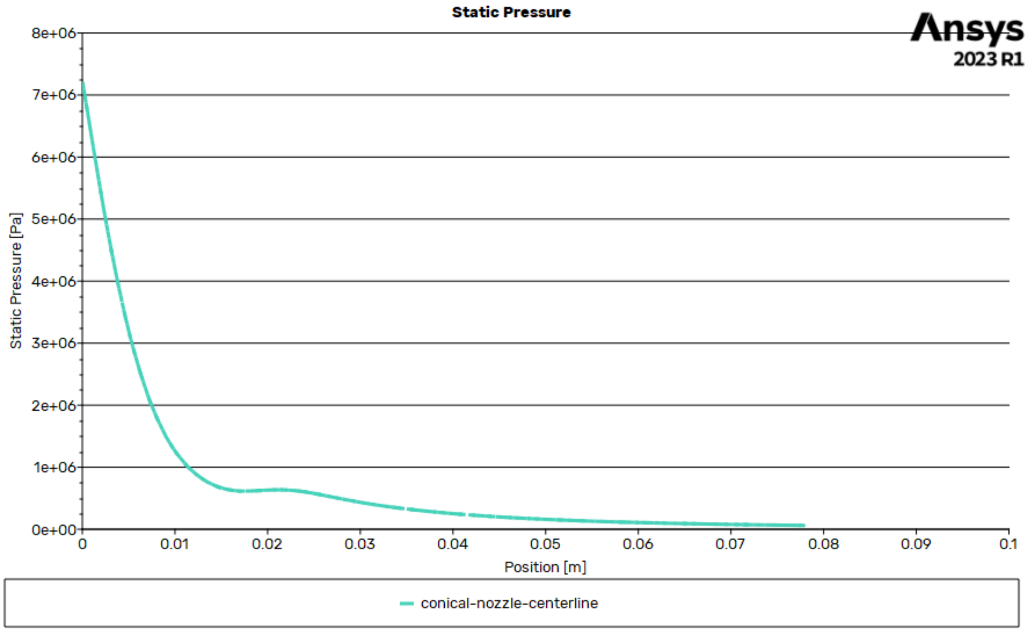

Figure 5: Static pressure distribution along the centerline of the conical nozzle.

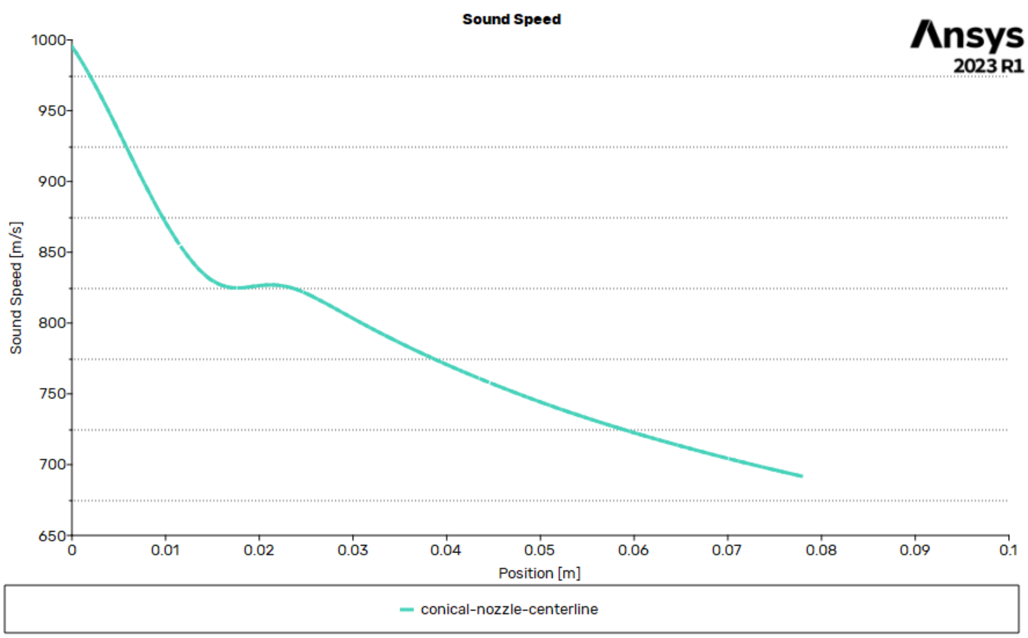

Figure 6: Speed of sound variation along the centerline of the conical nozzle.

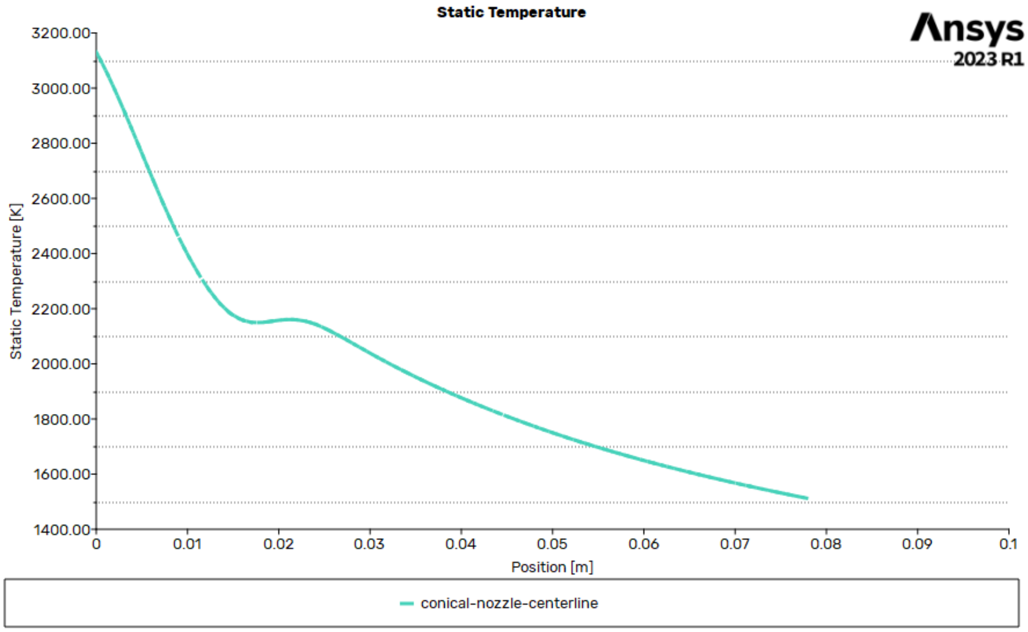

Figure 7: Static temperature distribution along the centerline of the conical nozzle.

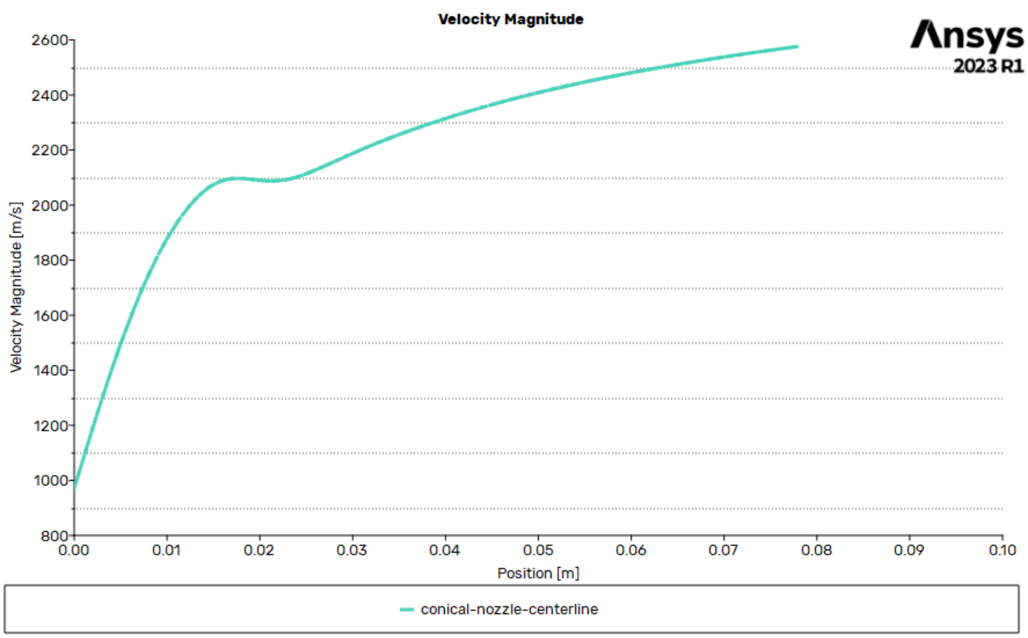

Figure 8: Velocity magnitude distribution along the centerline of the conical nozzle.

Conical Nozzle – Contour Plots

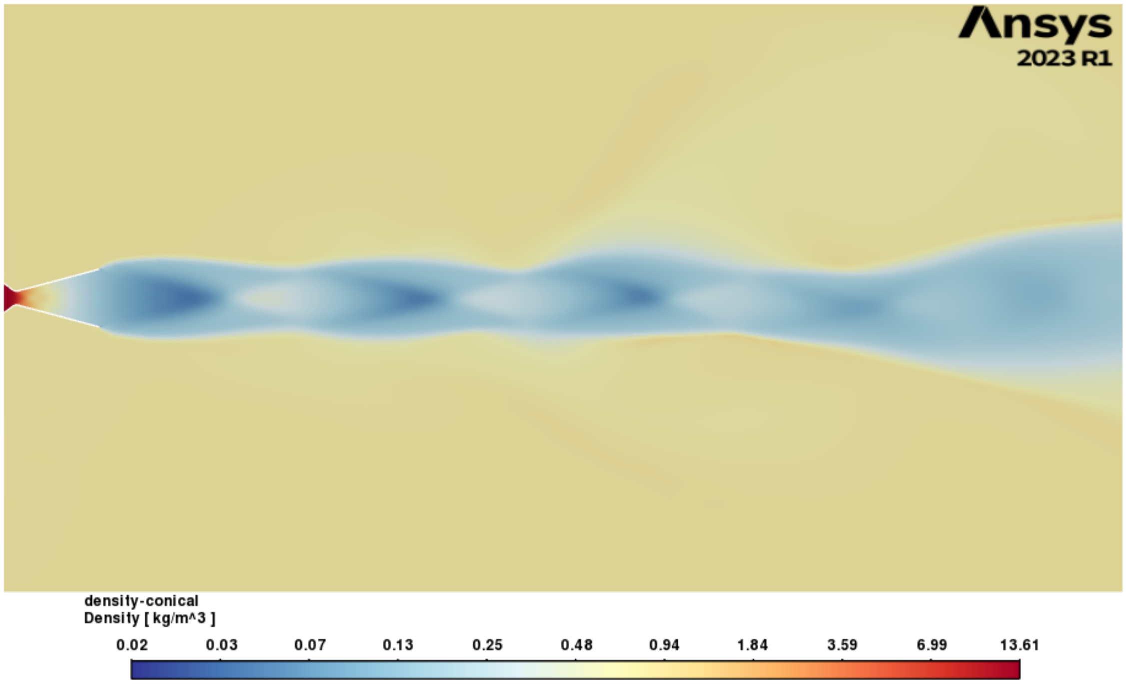

Figure 9: ANSYS simulation – Density contour in the conical nozzle.

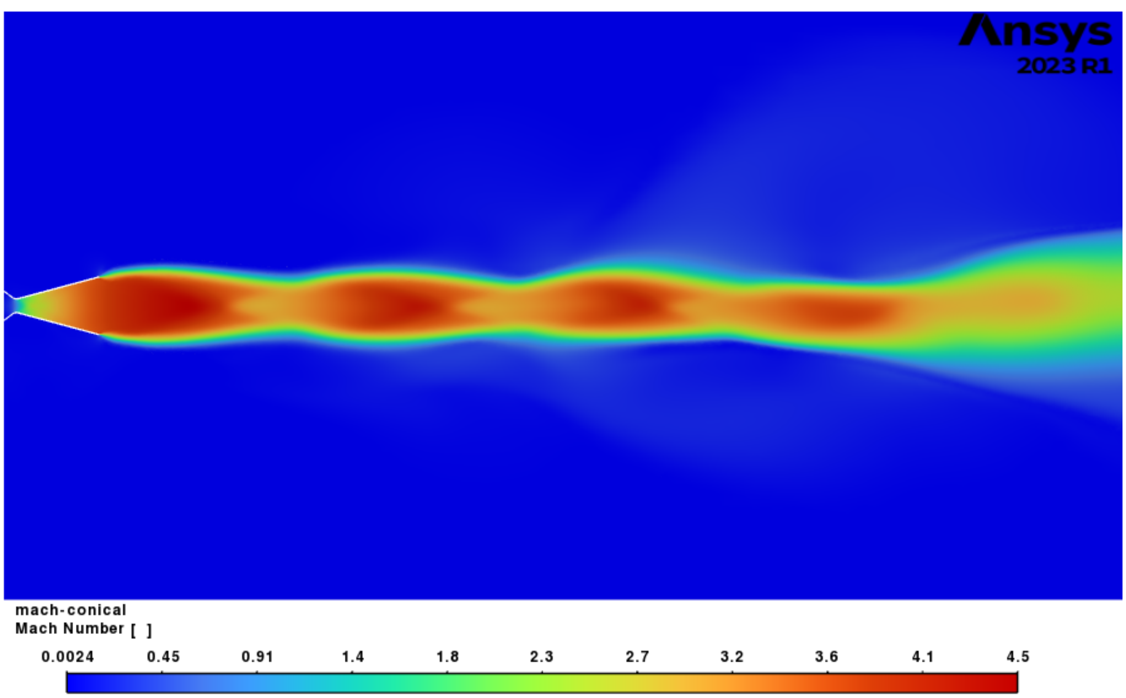

Figure 10: ANSYS simulation – Mach number contour in the conical nozzle.

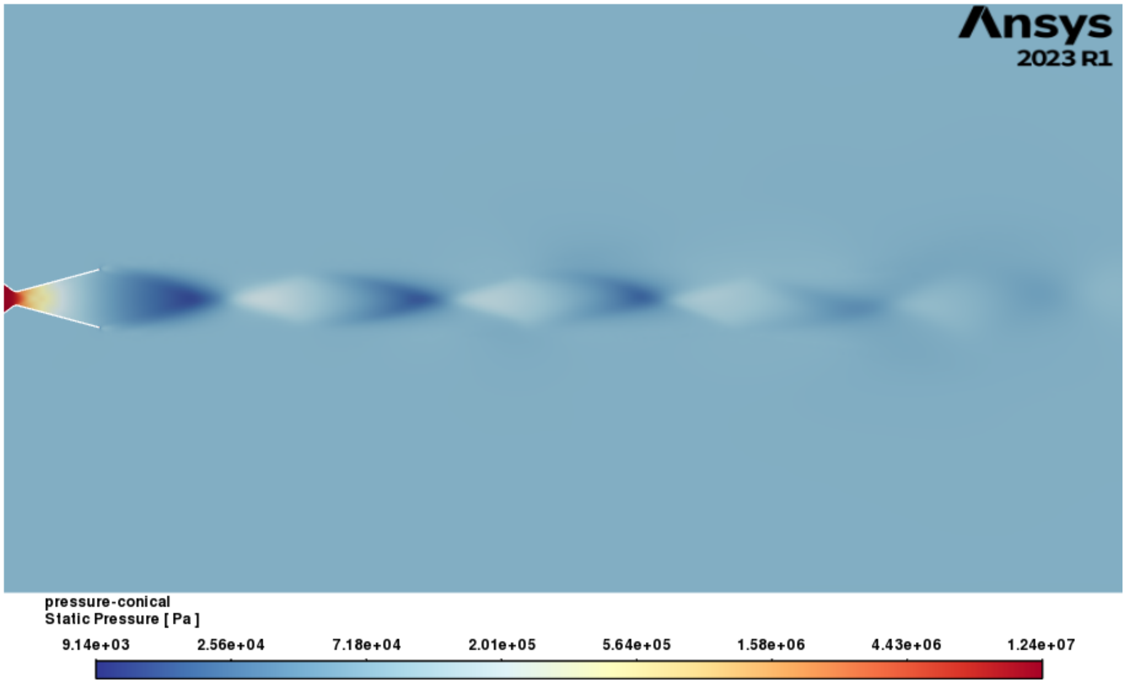

Figure 11: ANSYS simulation – Static pressure contour in the conical nozzle.

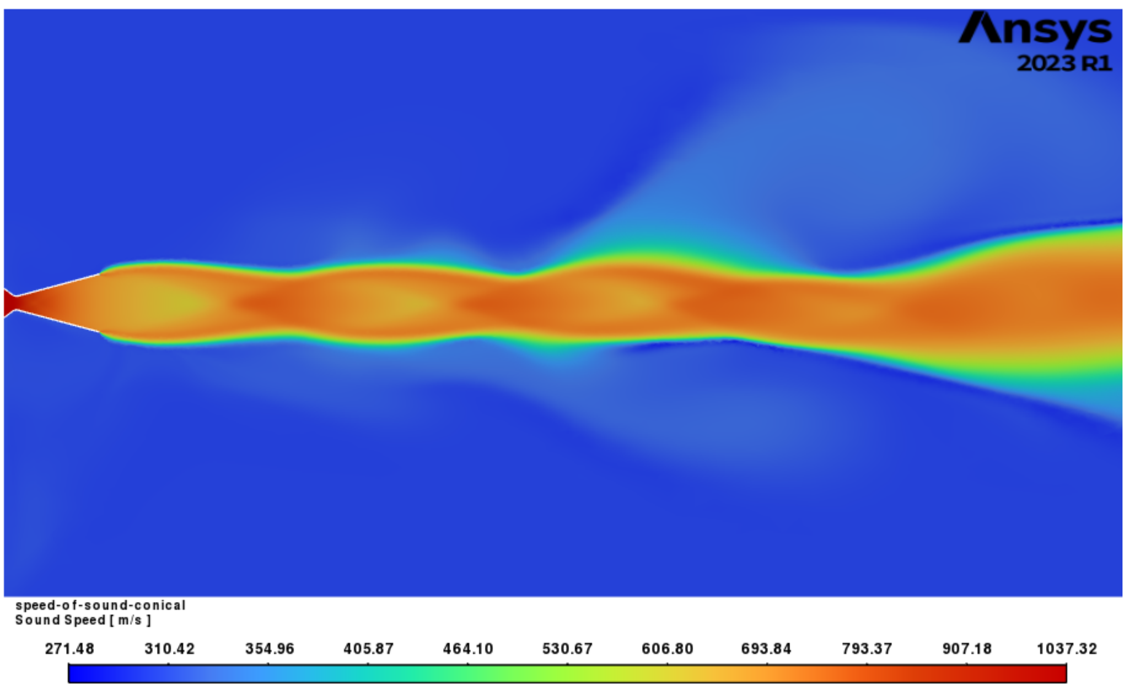

Figure 12: ANSYS simulation – Speed of sound contour in the conical nozzle.

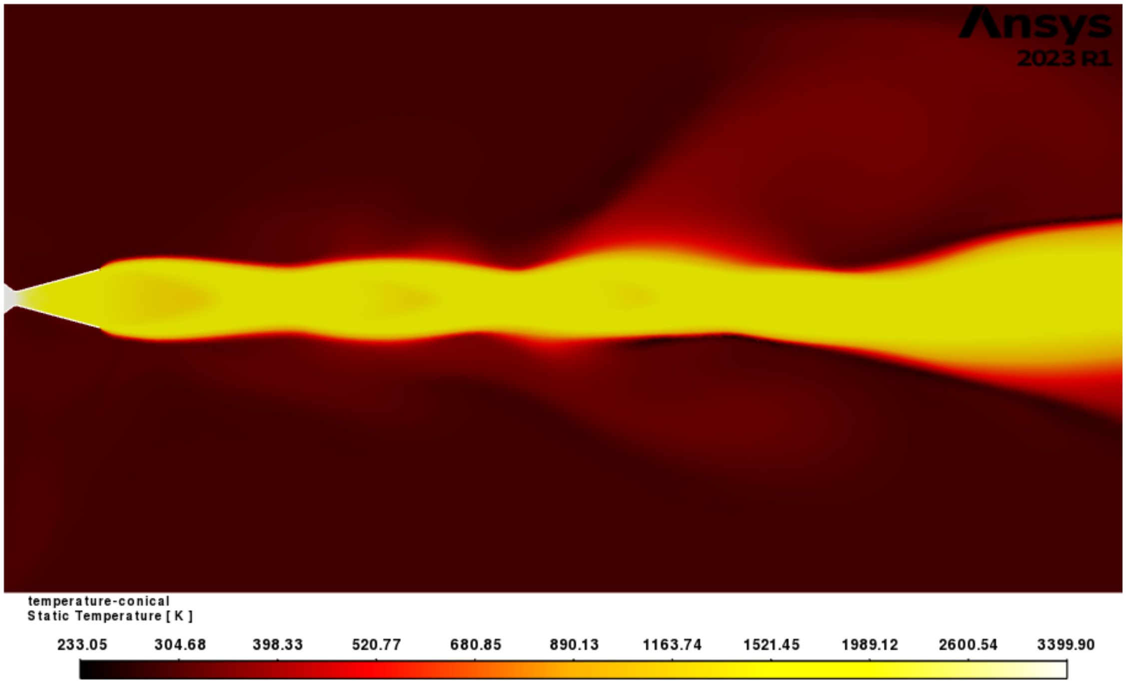

Figure 13: ANSYS simulation – Static temperature contour in the conical nozzle.

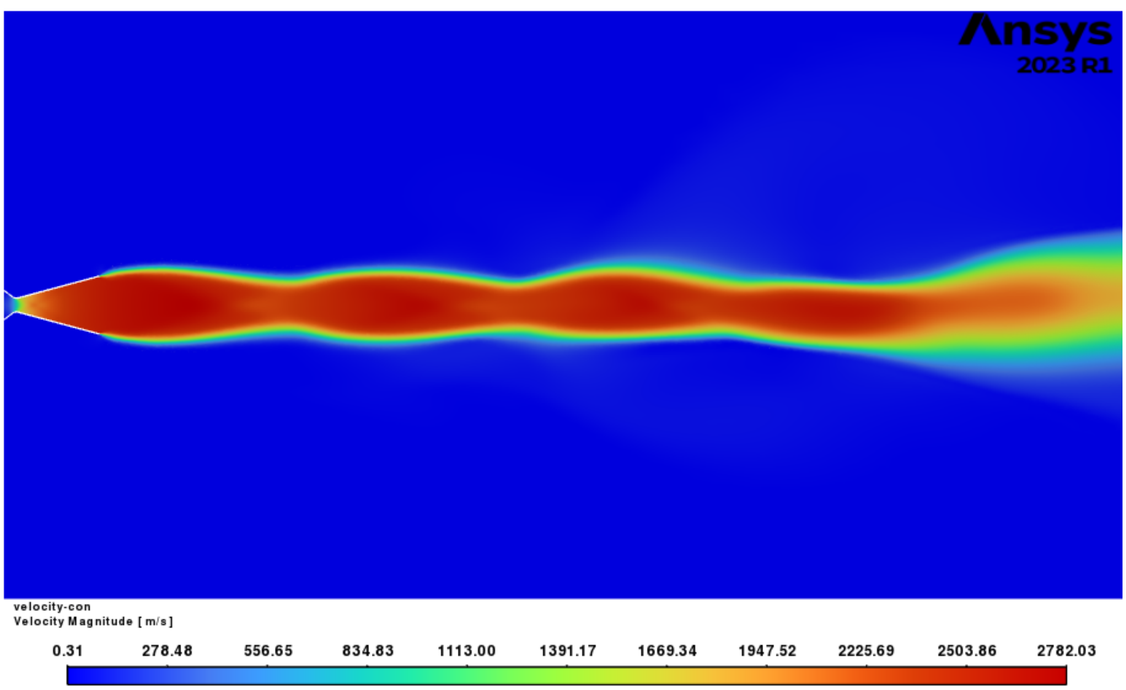

Figure 14: ANSYS simulation – Velocity magnitude contour in the conical nozzle.

Bell Nozzle – Line Plots

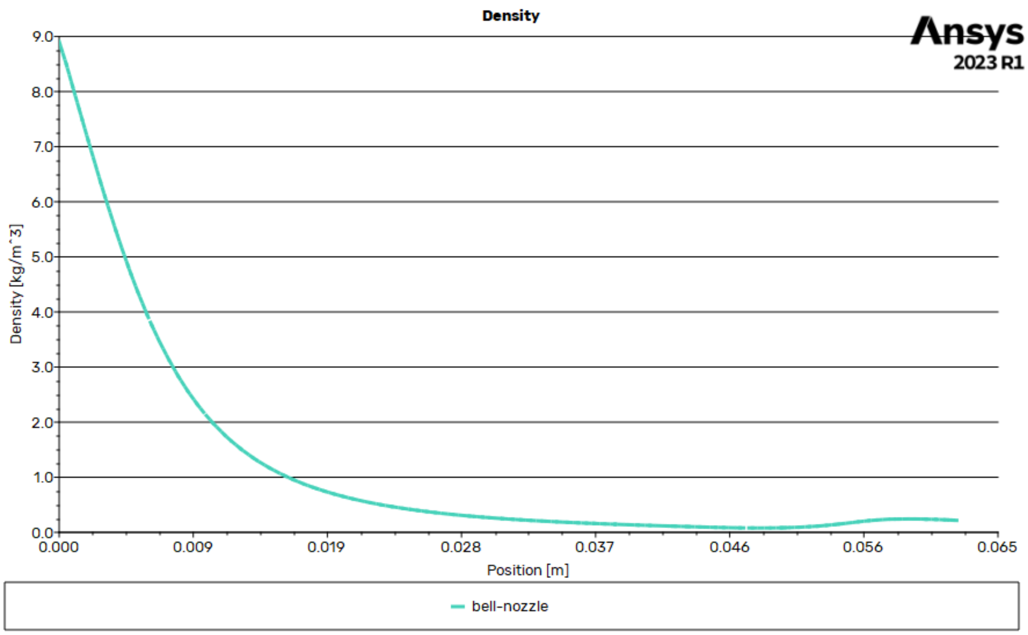

Figure 15: Density distribution along the bell nozzle.

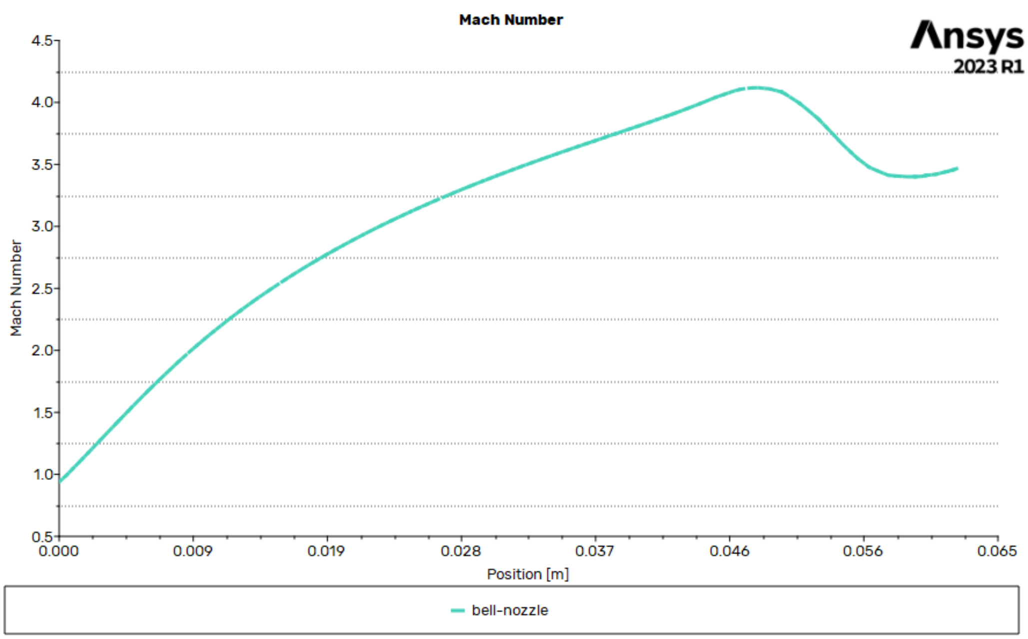

Figure 16: Mach number distribution along the bell nozzle.

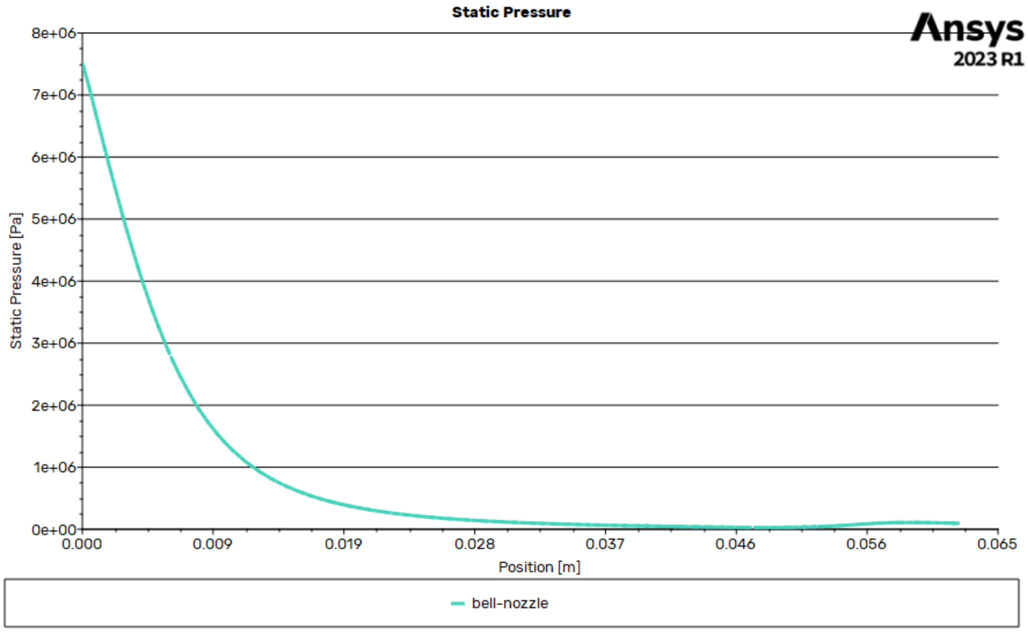

Figure 17: Static pressure distribution along the bell nozzle.

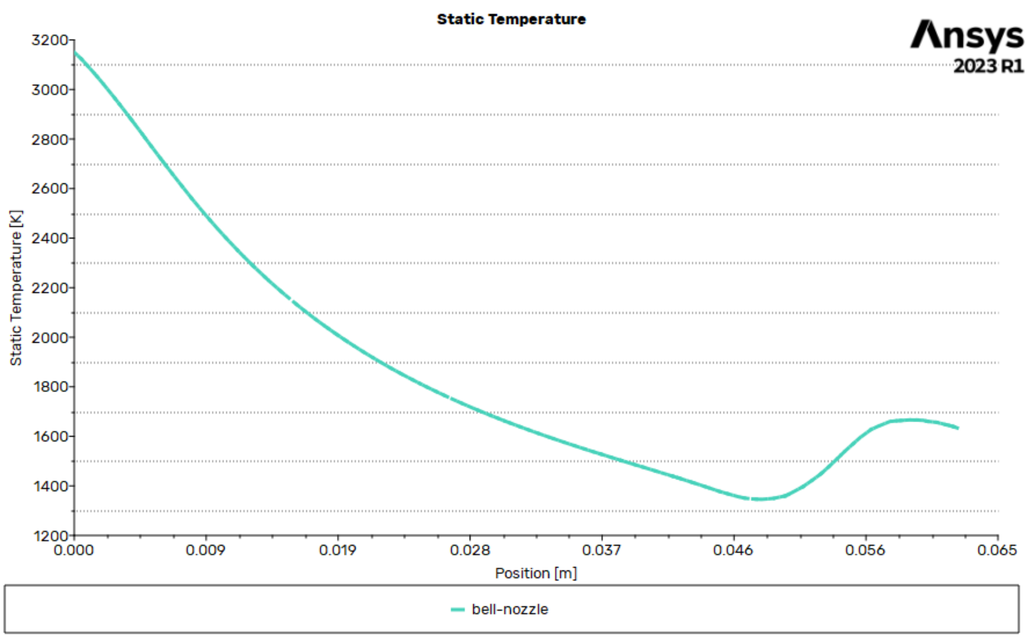

Figure 18: Static temperature distribution along the bell nozzle.

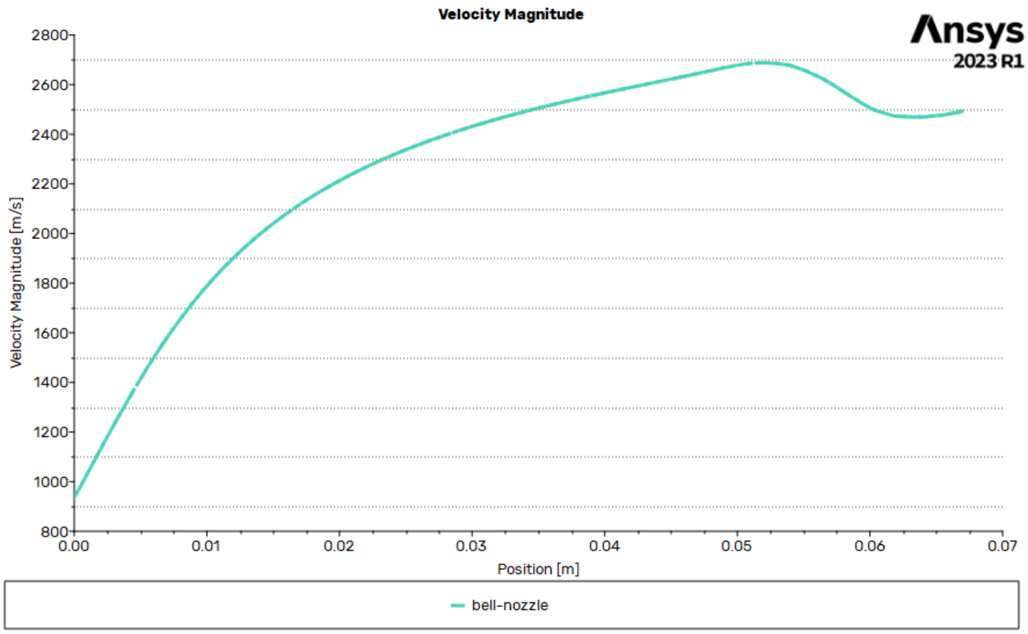

Figure 19: Velocity magnitude distribution along the bell nozzle.

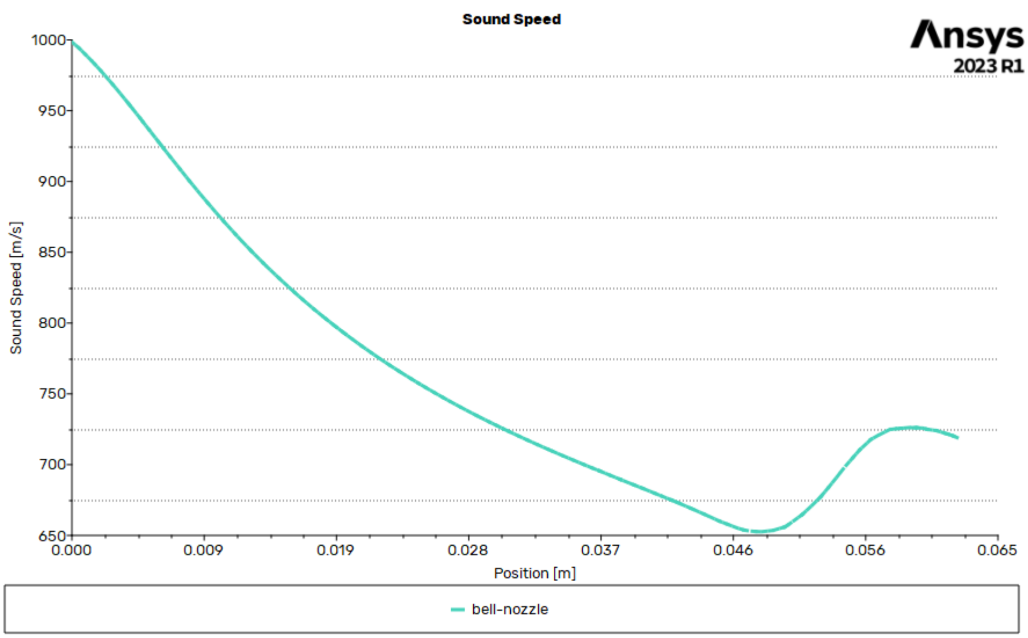

Figure 20: Speed of sound variation along the bell nozzle.

Bell Nozzle – Contour Plots

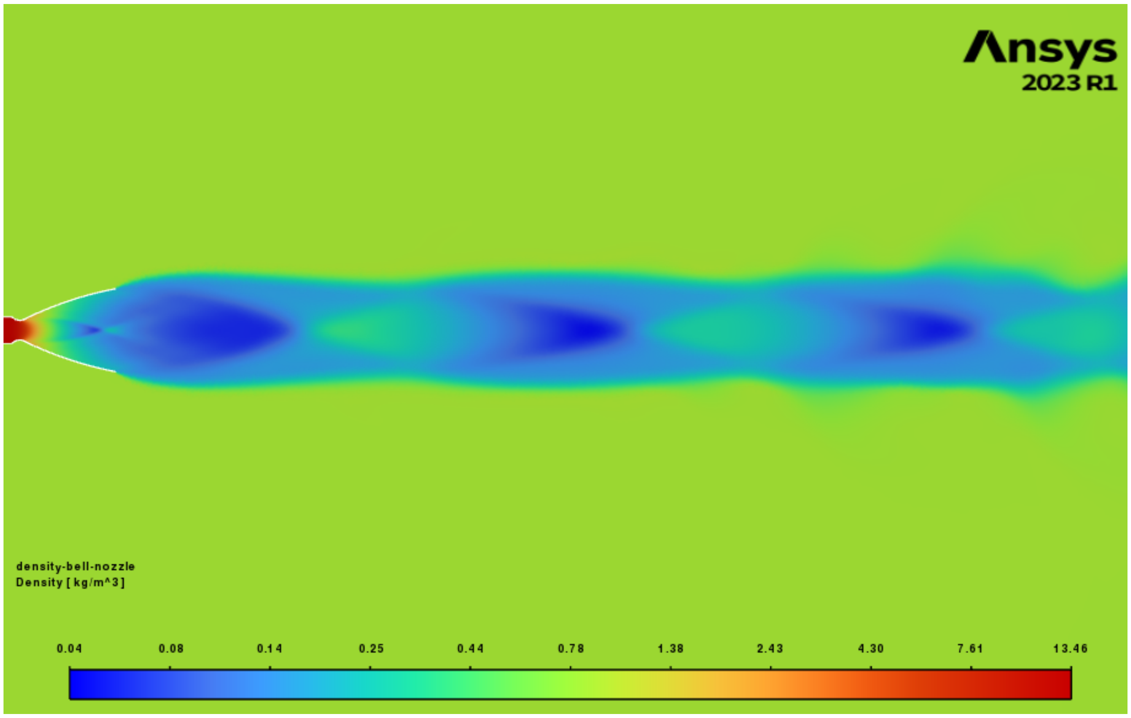

Figure 21: ANSYS simulation – Density contour in the bell nozzle.

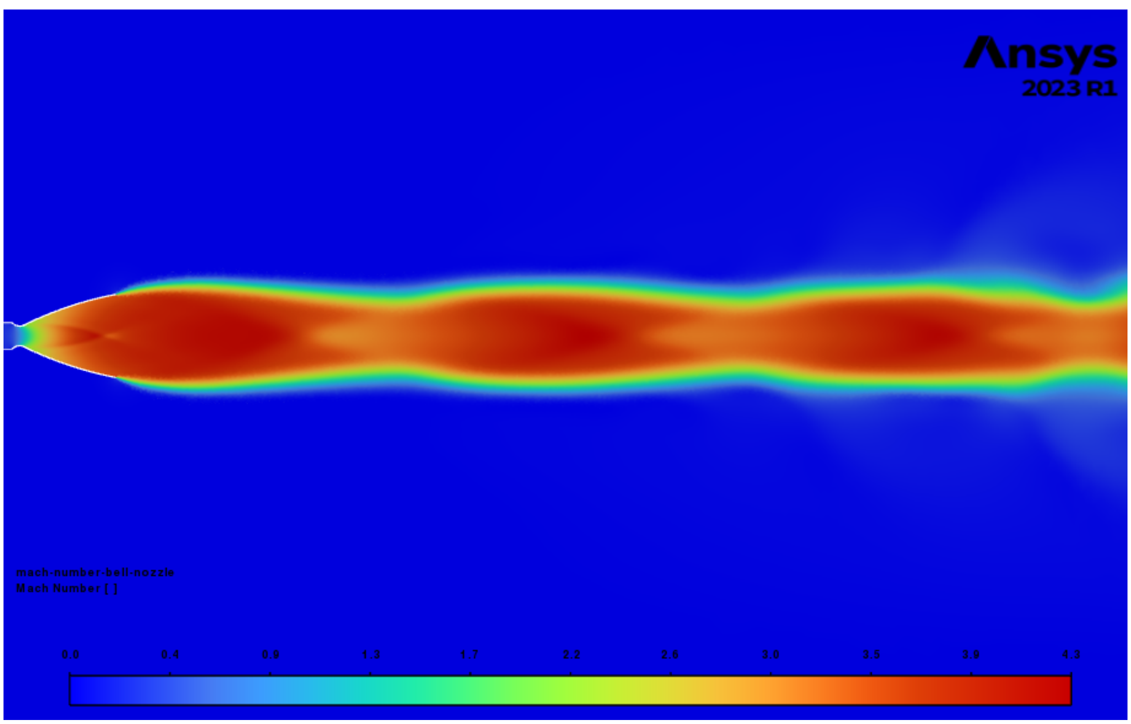

Figure 22: ANSYS simulation – Mach number contour in the bell nozzle.

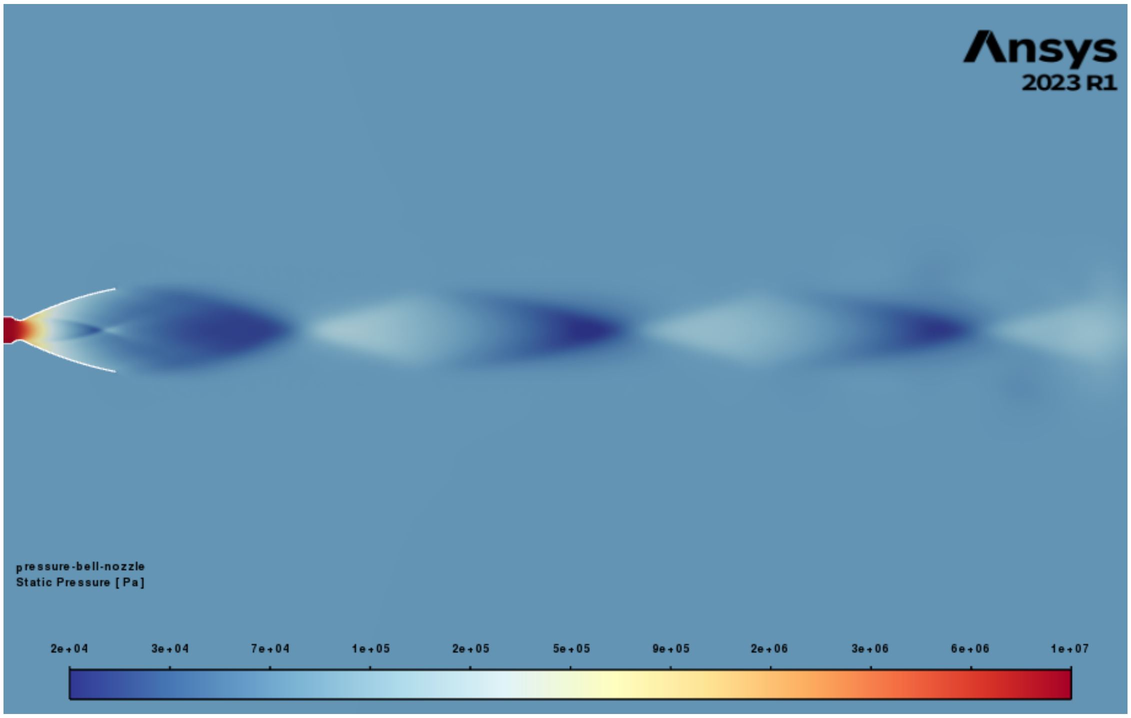

Figure 23: ANSYS simulation – Static pressure contour in the bell nozzle.

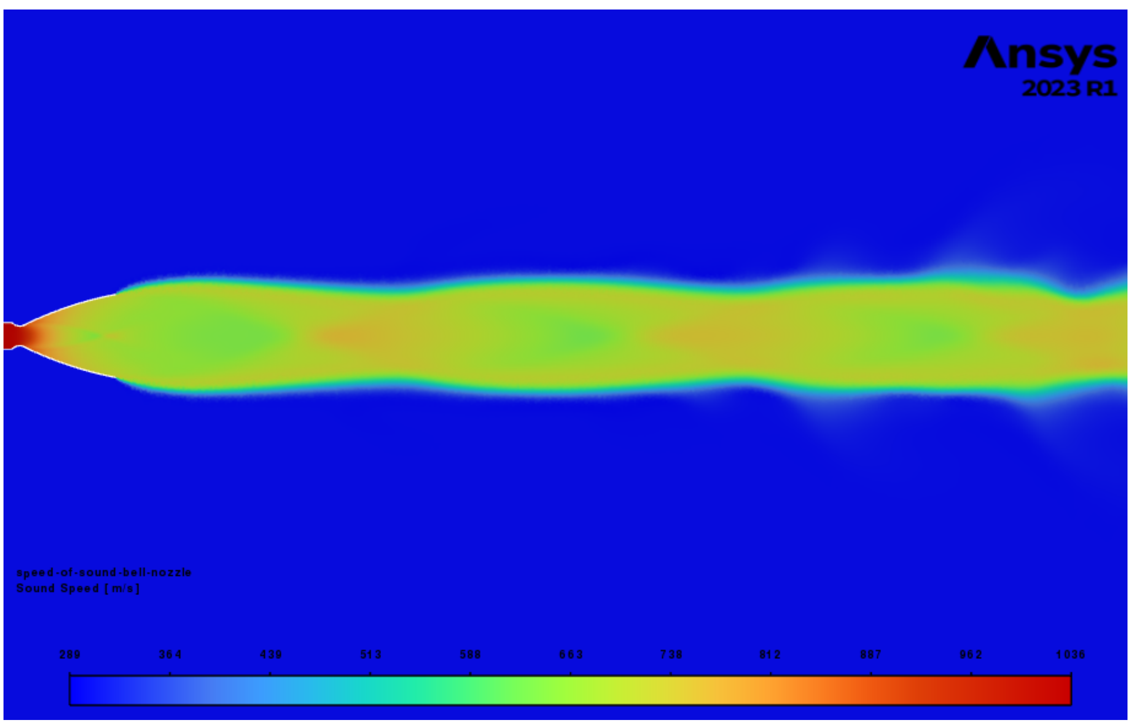

Figure 24: ANSYS simulation – Speed of sound contour in the bell nozzle.

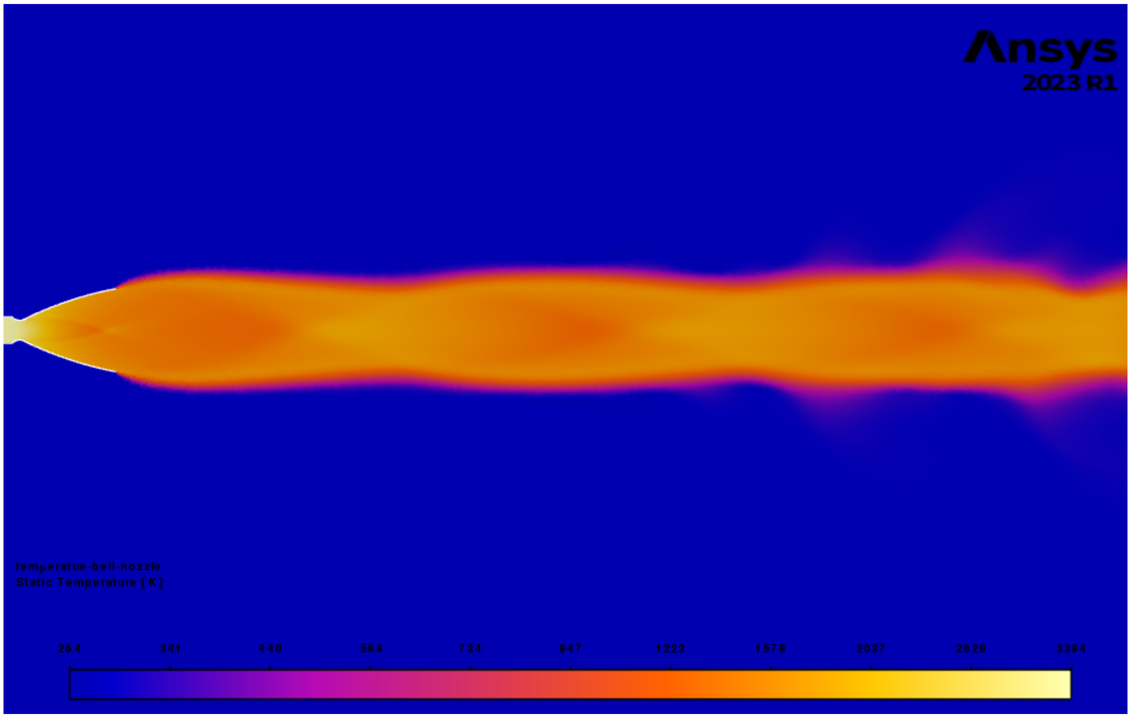

Figure 25: ANSYS simulation – Static temperature contour in the bell nozzle.

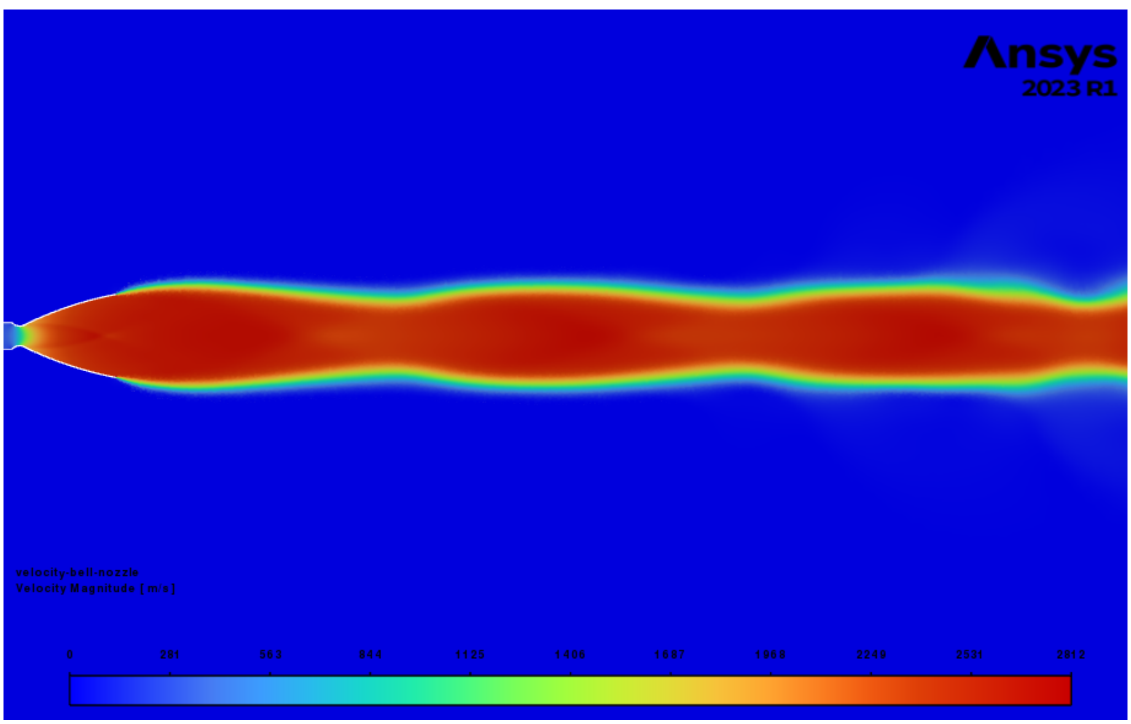

Figure 26: ANSYS simulation – Velocity magnitude contour in the bell nozzle.

C: OpenMotor Design

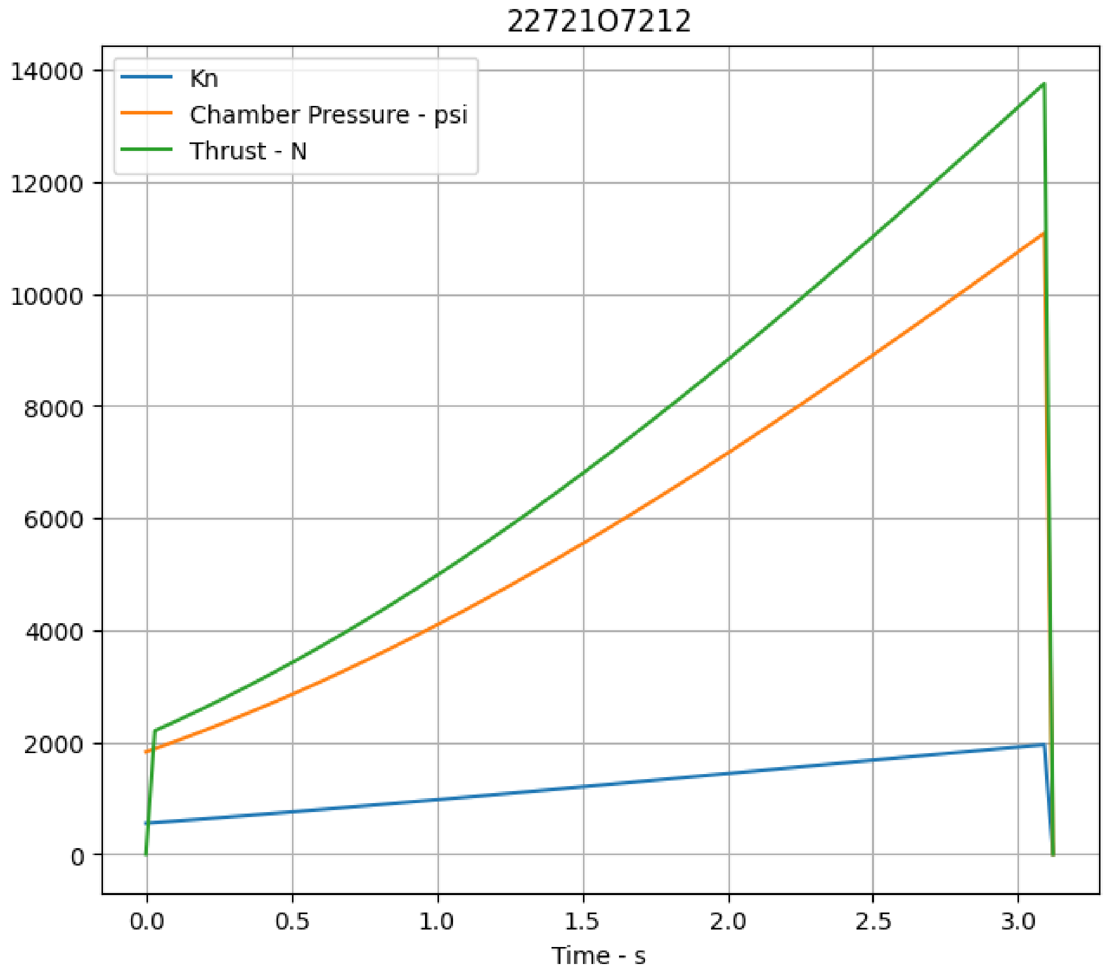

Figure 27: OpenMotor solid rocket motor design showing grain geometry and nozzle configuration.

Discussion: OpenMotor produces more realistic values as it accounts for grain regression, varying burning surface area, and nozzle flow effects dynamically throughout the burn. The theoretical values assume idealized steady conditions. Despite this, the percentage differences remain small: under 7% for burn time, and less than 2% for both impulse and I_{sp}. This indicates good agreement and validates the theoretical assumptions.

Quantity	**Theoretical**	**OpenMotor**	**Percent Difference**
Burn time (t_b)	2.93 s	3.12 s	6.47%
Specific impulse (I_{sp})	258.9 s	255.17 s	1.44%
Total impulse (I_{tot})	22,763.67 N·s	22,720.87 N·s	0.19%